

Teacher Key: What Does a Solution Mean?

Use and look-fors. Default to real-number solution sets in symbolic tasks and explicitly apply context restrictions. Distinguish a variable disappearing from a variable having value zero.

1. $2x = 12$, so $x = 6$. Check: $32 = 32$.
2. (a) All real x , since both sides equal $4x + 4$. (b) No solution: $4 = 5$ is false.
3. No solution. Subtracting $2x$ gives $7 = 9$, a contradiction. At zero the original equation is also false.
4. $\frac{1}{4}x = 2$, so $x = 8$. Check $4 + 3 = 2 + 5 = 7$.
5. $8 + 3n = 14 + 2n$, so $n = 6$ items; cost = 26 dollars. The point $(6, 26)$ belongs to both cost models.
6. $b = 6$. Otherwise subtracting $2x$ gives $6 = b$, false for every x .

Think Like a Mathematician

Extension solutions and teaching notes

7. Rewrite as $(a - 3)x = 2(a - 3)$. If $a \neq 3$, divide by the nonzero factor to obtain the single solution $x = 2$. If $a = 3$, the equation is an identity with all real x . No value of a gives no solution. Dividing first would illegally divide by zero in the identity case.

8. Agreement gives $3a + 4 = 6 + b$ and $7a + 4 = 14 + b$. Subtract to get $4a = 8$, so $a = 2$ and then $b = 4$. Both rules are then $2n + 4$, proving equality for every n .