

## Teacher Key: Ratios, Rates, and the Whole

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**Use and look-fors.** Assume ingredient volumes add and ribbon has no fixed fee. Develop page 2 common-unit reasoning before an efficient reciprocal procedure. Partial servings refer to a serving unit, not a liter. Page 4 is optional challenge.

1. Water: 3, 9, 15 cups. Total: 5, 15, 25 cups. Both ingredients scale together.
2.  $2 : 5$ ; 40%. The whole is all five cups of mixture.
3. No.  $\frac{2}{3} \neq \frac{6}{7}$ . Scale by 3: use 6 cups concentrate and 9 cups water.
4. One serving contains 8 twelfths of a liter. The leftover is one of those 8 units, so it is  $\frac{1}{8}$  serving.
5.  $q = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$ . Multiplication by  $\frac{3}{2}$  makes the coefficient of  $q$  equal 1.
6. 3,  $\frac{3}{2}$ , and  $\frac{9}{8}$  servings. A fixed amount holds fewer copies of a larger serving.
7. 8 cups per batch; 3 batches. Use 9 cups concentrate and 15 cups water.
8.  $\frac{5}{6} \div \frac{3}{4} = \frac{10}{12} \div \frac{9}{12} = \frac{10}{9} = 1\frac{1}{9}$  servings. Check:  $\frac{10}{9} \times \frac{3}{4} = \frac{5}{6}$ .
9. \$6 per meter;  $\frac{5}{2} \times 6 = \$15$ . The rate compares dollars to meters.

## Think Like a Mathematician

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**Extension solutions.** Offer a strategic hint before a procedure; accept complete alternative arguments.

10. **Work backward using equal parts.** The ratio 2 : 3 becomes 2 : 4 because concentrate stays fixed. The added 4 cups are one part. Original concentrate:  $2(4) = 8$  cups; water:  $3(4) = 12$  cups. Check  $8 : 12 = 2 : 3$  and  $8 : 16 = 1 : 2$ . Original total: 20 cups; final total: 24 cups.

11. **Use a common measuring unit.** The ribbon has 24 sixteenths. Let  $n$  and  $m$  be positive whole-number counts of short and long pieces. Then  $2n + 3m = 24$ . The number  $m$  must be even and less than 8. Thus  $(n, m) = (9, 2), (6, 4), (3, 6)$ . Totals: 11, 10, 9 pieces; maximum 11. A table gives a Grade 6 solution without requiring formal algebra.

## Teacher Decisions and Follow-Up

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### Listen for

The learner names both quantities in a ratio and distinguishes a fraction of a liter from a fraction of one serving.

### Reduce support when

The learner explains a non-unit-divisor quotient with common units or a missing-factor equation, then uses an efficient procedure and checks it.

### Return to a model when

If a leftover amount is called the same fraction of a serving, rebuild one serving and compare the leftover with that amount.

### A short sequence to compare

Compare  $\frac{3}{4}$  divided by  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{2}{3}$ . The amount stays fixed; as serving size grows, fewer servings fit. Explain partial servings.

### Delayed check: 1–2 weeks later

A drink uses 3 cups concentrate to 2 cups water. What fraction of the mixture is concentrate? A separate  $\frac{5}{6}$ -liter container holds how many  $\frac{2}{3}$ -liter servings, including part of a serving?

### Delayed-check interpretation

Concentrate is  $\frac{3}{5}$  of the mixture.  $\frac{5}{6}$  divided by  $\frac{2}{3} = \frac{5}{6}$  divided by  $\frac{4}{6} = \frac{5}{4}$  servings. After one serving,  $\frac{1}{6}$  liter remains, which is  $\frac{1}{4}$  of a  $\frac{2}{3}$ -liter serving. Check:  $(\frac{5}{4})(\frac{2}{3}) = \frac{5}{6}$ .

Record accuracy, conceptual explanation, and independent transfer separately. Note prompts or models supplied. These are instructional cues, not fixed mastery thresholds. Use the separate delayed-check booklet for student copies.