

Teacher Key: Proportion Across Representations

Use and look-fors. Keep x as meters and y as dollars when connecting the table and graph. Distinguish rate of change from total-cost-to-length ratio. The percent tasks change the referent whole.

1. Costs 0, 3, 6, 9 dollars; $y = \frac{3}{2}x$.
2. Plot $(0, 0)$, $(2, 3)$, $(4, 6)$, $(6, 9)$; draw the ray $y = \frac{3}{2}x$, $x \geq 0$. Each square is 1 meter horizontally and 1 dollar vertically.
3. Scale $(2, 3)$ by $\frac{5}{2}$: $3 \times \frac{5}{2} = \$7.50$. Or $\frac{3}{2} \times 5 = \$7.50$.
4. At 2 meters, $y/x = 5/2$; at 4 meters, $y/x = 8/4 = 2$. Not constant. The graph has intercept 2; a constant slope does not make total cost proportional to length.
5. $k = \frac{3}{8}$ cm/km; $y = \frac{3}{8}x$; 16 km gives 6 cm.
6. $48(0.75) = 36$; $36(1.05) = \$37.80$. A single 20% discount gives \$38.40. The tax uses \$36 as its referent amount.

Think Like a Mathematician

Extension solutions. Offer a strategic hint before a procedure; accept complete alternative arguments.

7. Multipliers $\frac{4}{5}$ and $\frac{11}{10}$ give $\frac{22}{25} = 0.88$, a 12% decrease. Original price: $39.60/0.88 = \$45$. Check $45(0.8)(1.1) = 39.60$. Reversing the order preserves the product under exact calculations. Adding the percent changes ignores their different bases.

8. Original rate: $\frac{3}{8}$ cm/km. Enlargement: $\frac{3}{8} \times \frac{8}{5} = \frac{3}{5}$ cm/km, so 9 cm represents $9 \div \frac{3}{5} = 15$ km.

Combined size factor: $\frac{8}{5} \times \frac{3}{5} = \frac{24}{25} = 96\%$. The final map is 4% smaller in each linear dimension. Its rate is $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$ cm/km.

Teacher Decisions and Follow-Up

Listen for

The learner connects a labeled table, graph, and equation and distinguishes a constant ratio from a constant rate of change.

Reduce support when

The learner tests a new relationship using units, a nonzero input ratio, and the starting value, then justifies a prediction independently.

Return to a model when

If every straight line is called proportional, compare two coordinate ratios and the value at zero.

A short sequence to compare

Compare $y = (\frac{3}{2})x$ with $y = (\frac{3}{2})x + 2$ at $x = 0, 2$, and 4 . What is unchanged? What prevents the second relationship from being proportional?

Delayed check: 1–2 weeks later

A trail map uses $\frac{2}{3}$ centimeter for 4 kilometers. Write map length y in terms of actual distance x and find the map length for 15 kilometers. Is $y = x/6 + 1$ proportional? Explain.

Delayed-check interpretation

$k = (\frac{2}{3})/4 = \frac{1}{6}$ cm/km, so $y = x/6$ and $y = \frac{5}{2} = 2.5$ cm for 15 km. The added-1 relationship is not proportional: its intercept is 1 and y/x is not constant.

Record accuracy, conceptual explanation, and independent transfer separately. Note prompts or models supplied. These are instructional cues, not fixed mastery thresholds. Use the separate delayed-check booklet for student copies.