

Teacher Key: Fractions in Linear Relationships

Use and look-fors. Page 1 addresses Grade 8 linear relationships; page 2 develops Algebra I equivalent equations. The tank model applies while filling. Rental time is continuous with no rounding fees.

1. $m = \frac{6-3}{4-0} = \frac{3}{4}$; $y = \frac{3}{4}x + 3$. The 3 is the starting value at $x = 0$.

2. $6/4$ compares coordinates from the origin, which is not on the line. Changes between the given points give $3/4$.

3. $V(t) = 5 + \frac{3}{4}t$ while filling, $t \geq 0$. $V(8) = 11$ liters. Not proportional because the initial amount is 5 liters.

4. Isolate: $\frac{2}{3}x = 6$, $x = 9$. Clear: $2x + 3 = 21$, $x = 9$. Check $\frac{2}{3}(9) + 1 = 7$.

5. The constant must also be doubled twice: $3x + 8 = 32$, so $x = 8$. Check $\frac{3}{4}(8) + 2 = 8$.

6. $C(h) = 6 + \frac{5}{2}h$. For 21 dollars, $\frac{5}{2}h = 15$ gives $h = 6$. Or $12 + 5h = 42$, so $h = 6$. The \$6 is a fixed fee.

Think Like a Mathematician

Extension solutions. Offer a strategic hint before a procedure; accept complete alternative arguments.

7. $6 + \frac{5}{2}h = 9 + \frac{7}{4}h$ gives $h = 4$ hours and a cost of \$16. Plan A is cheaper for $0 \leq h < 4$; B is cheaper for $h > 4$. A starts \$3 cheaper but gains cost at $\frac{3}{4}$ dollar more per hour. Closing the gap takes $3 \div \frac{3}{4} = 4$ hours. The difference $C_A - C_B = \frac{3}{4}h - 3$ confirms the intervals.

8. Slope $\frac{(a+3)-a}{6-2} = \frac{3}{4}$. Substitution gives $b = a - \frac{3}{2}$. Proportionality requires $b = 0$, so $a = \frac{3}{2}$. Then $y = \frac{3}{4}x$, and $y(10) = \frac{15}{2}$. Changing a shifts all y-values equally and preserves slope. Grade 8 students may compare numerical cases before the parameter argument.

Teacher Decisions and Follow-Up

Listen for

The learner uses changes between two points for slope and preserves equality when operating on an equation.

Reduce support when

The learner translates a new context to a linear equation, interprets both coefficients, and checks a solution by substitution or another method.

Return to a model when

If y/x is used for a line with an intercept, draw a slope triangle between two points on that line. If clearing denominators loses a term, mark every term on both sides.

A short sequence to compare

Compare $y = (\frac{3}{2})x$ and $y = (\frac{3}{2})x + 4$ at $x = 0$ and 2 . Their changes match even though their starting values differ.

Delayed check: 1–2 weeks later

A line passes through $(2,5)$ and $(6,8)$. Find its slope and equation, then find x when $y = 11$. Explain why $\frac{5}{2}$ is not its slope.

Delayed-check interpretation

$m = (8-5)/(6-2) = \frac{3}{4}$. Using $(2,5)$, $b = \frac{7}{2}$, so $y = (\frac{3}{4})x + \frac{7}{2}$. For $y = 11$, $x = 10$. Check: $\frac{15}{2} + \frac{7}{2} = 11$. The ratio $\frac{5}{2}$ compares coordinates from the origin, which is not on this line.

Record accuracy, conceptual explanation, and independent transfer separately. Note prompts or models supplied. These are instructional cues, not fixed mastery thresholds. Use the separate delayed-check booklet for student copies.