

Fraction Fix

Teacher Guide and Answer Key · Lessons 11–15

Purpose This block establishes fractions as precisely located numbers and develops efficient magnitude comparisons. Students move from exact partitioning to benchmark estimation, then compare same-denominator and same-numerator cases with explicit conditions.

Instructional throughline Establish 0, 1, and nearby wholes; name the unit length; estimate before partitioning; and require every shortcut to include the condition that makes it valid.

Evidence across Lessons 11–15

Dimension	Evidence
Scale	Uses 0, 1, and nearby wholes to determine interval length.
Location	Iterates unit fractions from zero, including beyond one.
Magnitude	Estimates with benchmarks and checks reasonableness.
Comparison	Uses same-unit or same-count reasoning under valid conditions.
Transfer	Diagnoses an off-by-one or whole-number comparison error.

Pacing note Each lesson is designed for 50 minutes. Preserve the estimate and explanation even if one practice item must move to the next day’s retrieval.

Instructional commitments

- Establish the scale before placing a fraction.
- Ask for a magnitude estimate before exact partitioning.
- Require the condition that makes a comparison shortcut valid.
- Use errors to identify the student’s current model, then select a representation that tests it.

Lesson 11 | Locating Proper Fractions

Lesson aim	Materials	Evidence to collect
Locate proper fractions by iterating equal unit lengths from zero.	Workbook; rulers; number-line strips	Students establish scale, count intervals, and justify a point with a unit fraction and benchmark.

Suggested 50-minute arc

Minutes	Teacher move
5	Retrieve scale and intervals.
8	Estimate $\frac{2}{3}$ before partitioning.
12	Build stacked number lines.
10	Formalize $\frac{a}{b}$ as a intervals of length $\frac{1}{b}$.
10	Practice and diagnose off-by-one reasoning.
5	Exit ticket.

Facilitation notes

- Require students to identify 0 and 1 before adding partitions.
- Treat the unit fraction as a length, not merely a label at a tick mark.
- Ask for estimates first so partitioning confirms a magnitude claim.
- An off-by-one error is evidence that the student is counting points rather than intervals.

Answer key and look-fors

- Five intervals; $\frac{3}{5}$ at the third endpoint.
 - Denominators name equal interval lengths; tick marks are boundaries. 3–4. $\frac{2}{3}$ is greater than $\frac{1}{2}$ and less than 1.
 - Correct placements: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.
 - $\frac{3}{4}$ is closest to 1.
 - $\frac{5}{8}$ is the fifth eighth-length step.
 - Partition 0–1 into b equal intervals and travel a intervals from 0.
 - Order: $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{6}$.
 - $\frac{2}{5}$ is the second endpoint after 0, not the second tick when 0 is counted.
 - $\frac{4}{7}$ is $\frac{1}{14}$ above $\frac{1}{2}$ and $\frac{3}{7}$ below 1, so it is closer to $\frac{1}{2}$.
- Exit: $\frac{2}{3}$ at the second of three equal intervals.

Lesson 12 | Locating Improper Fractions

Lesson aim	Materials	Evidence to collect
Extend a constant fractional unit across whole-number boundaries.	Workbook; rulers; extended number-line strips	Students locate and bound improper fractions without converting first.

Suggested 50-minute arc

Minutes	Teacher move
5	Retrieve whole numbers written in fourths.
8	Count by fourths through $11/4$.
12	Locate thirds and decompose by wholes.
10	Use whole-number bounds efficiently.
10	Diagnose the “all fractions below 1” claim.
5	Exit ticket.

Facilitation notes

- Keep every interval the same length after the line passes 1.
- Use $4/4$, $8/4$, and $12/4$ as named whole-number benchmarks.
- Mixed-number language may appear, but location should be reasoned from repeated units.
- Ask students to measure distance from 2 in fractional units.

Answer key and look-fors

1. Four fourths; $1 = 4/4$ and $2 = 8/4$.
 2. $7/4$ is between 1 and 2.
 3. Labels increase by $1/4$ through $11/4$.
 4. The unit length remains $1/4$.
 5. $11/4$ is farthest from 2 among the three named points.
 6. $5/3$ lies between 1 and 2; $8/3$ between 2 and 3.
 7. $5/3 = 3/3 + 2/3$; $8/3 = 3/3 + 3/3 + 2/3$; $13/5 = 5/5 + 5/5 + 3/5$; $10/4 = 4/4 + 4/4 + 2/4$.
 8. Divide the numerator by the denominator; the quotient counts complete wholes.
 9. $13/5 = 2 \frac{3}{5}$.
 10. $7/3$ is the seventh third-length step, between 2 and 3.
 11. Answers include $13/6$ through $17/6$.
 12. $7/4$ is $1/4$ from 2; $10/4$ is $2/4$ from 2, so $10/4$ is farther.
- Exit: $8/3$ lies between 2 and 3 and equals $3/3 + 3/3 + 2/3$.

Lesson 13 | Fraction Number Line Estimation

Lesson aim	Materials	Evidence to collect
Use benchmark numbers to estimate fraction locations efficiently.	Workbook; open number lines; optional fraction strips	Students bound fractions and refine placement without partitioning every interval.

Suggested 50-minute arc

Minutes	Teacher move
5	Retrieve whole-number bounds.
8	Build half-unit benchmarks from 0 to 3.
12	Estimate $\frac{7}{8}$, $\frac{5}{4}$, and $\frac{13}{8}$.
10	Sort and place eight fractions.
10	Diagnose and refine unreasonable points.
5	Exit ticket.

Facilitation notes

- Accept approximate points when the supporting interval and relative order are correct.
- Ask which benchmark did the most work; do not reward needless partitioning.
- Use numerator-denominator relationships: equal, half, almost equal, or more than one whole.
- A precise estimate includes a defensible interval, not a visually perfect dot.

Answer key and look-fors

1. $\frac{13}{8}$ lies between 1 and 2.
 2. Examples: $\frac{2}{4}$ and $\frac{4}{2}$.
 3. Half-unit labels from 0 through 3.
 4. $\frac{7}{8}$ just left of 1; $\frac{5}{4}$ halfway between 1 and $1\frac{1}{2}$; $\frac{13}{8}$ just right of $1\frac{1}{2}$.
 6. 0– $\frac{1}{2}$: $\frac{1}{10}$, $\frac{4}{9}$; $\frac{1}{2}$ –1: $\frac{7}{10}$; 1– $1\frac{1}{2}$: $\frac{9}{8}$; $1\frac{1}{2}$ –2: $\frac{7}{4}$, $\frac{15}{8}$; 2– $2\frac{1}{2}$: $\frac{11}{5}$; $2\frac{1}{2}$ –3: $\frac{17}{6}$.
 7. Order follows those values.
 10. $\frac{13}{8}$ is greater than $\frac{12}{8} = 1\frac{1}{2}$ and less than $\frac{16}{8} = 2$.
 11. $\frac{3}{2} < \frac{17}{10} < \frac{7}{4}$ is an interval of width $\frac{1}{4}$.
 12. $\frac{23}{12}$ is $\frac{1}{12}$ below 2 and much closer to 2 than 1.
- Exit: $\frac{13}{8}$ lies between $1\frac{1}{2}$ and 2.

Lesson 14 | Same Denominator Comparisons

Lesson aim	Materials	Evidence to collect
Compare counts of a common unit and state the condition for the shortcut.	Workbook; number-line strips	Students compare and order same-denominator fractions using unit-fraction language.

Suggested 50-minute arc

Minutes	Teacher move
5	Retrieve location and unit names.
8	Place $\frac{3}{8}$, $\frac{7}{8}$, and $\frac{11}{8}$.
12	Generalize same-denominator reasoning.
10	Practice comparisons and ordering.
10	Test the claim with a counterexample.
5	Exit ticket.

Facilitation notes

- Require the same referent whole as well as the same denominator.
- Say “more copies of one eighth,” not merely “the numerator is larger.”
- Include values above one so the strategy is not tied to proper fractions.
- Use the counterexample to establish that numerator comparison is conditional.

Answer key and look-fors

1. $\frac{5}{8}$ lies between 0 and 1; $\frac{11}{8}$ between 1 and 2.
 2. Units are $\frac{1}{10}$.
 3. Correct placements in increasing numerator order.
 4. The unit $\frac{1}{8}$ stays constant; the number of copies changes.
 5. larger; more.
 6. $\frac{5}{12} < \frac{9}{12}$; $\frac{14}{9} > \frac{11}{9}$; $\frac{7}{5} > \frac{5}{5}$; $\frac{18}{20} < \frac{19}{20}$.
 7. $\frac{6}{7} < \frac{7}{7} < \frac{13}{7} < \frac{15}{7}$.
 8. $\frac{12}{16}$ m is longest and exceeds $\frac{5}{16}$ m by $\frac{7}{16}$ m.
 9. Answers vary, e.g., $\frac{12}{11} > \frac{10}{11}$.
 10. Works: $\frac{7}{9} > \frac{4}{9}$. Fails: $\frac{3}{10} < \frac{2}{3}$.
 11. $\frac{11}{8} > \frac{7}{8}$ because eleven eighths are more copies of $\frac{1}{8}$ than seven eighths.
 12. Examples: $\frac{7}{12}$ and $\frac{2}{12}$.
- Exit: $\frac{11}{8} > \frac{7}{8}$.

Lesson 15 | Same Numerator Comparisons

Lesson aim	Materials	Evidence to collect
Compare equal counts of differently sized unit fractions.	Workbook; fraction strips or paper bars	Students use unit size to compare same-numerator fractions and reject whole-number reasoning.

Suggested 50-minute arc

Minutes	Teacher move
5	Retrieve same-denominator comparison.
8	Predict $\frac{3}{5}$ versus $\frac{3}{8}$.
12	Represent equal counts of fifths and eighths.
10	Generalize the same-numerator condition.
10	Practice, diagnose, and transfer.
5	Exit ticket.

Facilitation notes

- Use identical wholes for the visual comparison.
- Hold the copy count constant and focus attention on unit size.
- Do not introduce cross-products here; preserve the magnitude argument.
- The denominator-10 creation task should reveal that the comparison rule depends on the other denominator, not on 10 alone.

Answer key and look-fors

1. $\frac{7}{9} > \frac{4}{9}$.
 2. $\frac{1}{6} > \frac{1}{9}$ for the same whole. 3–4. $\frac{3}{5} > \frac{3}{8}$ because fifths are larger units than eighths.
 5. smaller; larger.
 6. $\frac{5}{6} > \frac{5}{9}$; $\frac{7}{12} < \frac{7}{10}$; $\frac{4}{5} > \frac{4}{7}$; $\frac{11}{8} > \frac{11}{12}$.
 7. $\frac{6}{13} < \frac{6}{11} < \frac{6}{8} < \frac{6}{7}$.
 8. $\frac{5}{8}$ is longer.
 9. Physical quantities cannot be compared unless the wholes match.
 10. $\frac{5}{9} < \frac{5}{6}$ because ninths are smaller than sixths.
 11. $\frac{5}{6}$ is $\frac{1}{6}$ from 1; $\frac{5}{9}$ is $\frac{4}{9}$ from 1.
 12. Example: $\frac{3}{10} > \frac{3}{12}$. The other denominator must exceed 10.
- Exit: $\frac{5}{6} > \frac{5}{9}$ because six equal parts are larger than nine equal parts of the same whole.