

Fraction Fix

Teacher Guide | Lessons 31-35

Meaning first. Efficient procedures from understanding. Durable fluency through retrieval.

Lesson progression

- 31 Subtracting Unlike Fractions**
- 32 Mixed Number Addition and Subtraction**
- 33 Fraction Times a Whole Number**
- 34 A Fraction of a Fraction**
- 35 Deriving the Multiplication Procedure**

Use each guide page to protect the conceptual throughline, anticipate misconceptions, and collect exit evidence.

Lesson 31 | Subtracting Unlike Fractions

Teacher Guide

Instructional focus

Interpret subtraction as take-away and distance, then compute with common units.

Primary representation: Number line and strips

Suggested 50-minute arc

5 min	Cumulative retrieval and estimate
8 min	Launch the central comparison or context
12 min	Student modeling and partner reasoning
10 min	Connect representation, language, and notation
10 min	Focused practice with immediate feedback
5 min	Exit evidence: computation plus explanation

Facilitation notes

- Ask students to estimate or predict before applying a procedure.
- Require the referent whole and the fractional unit to remain explicit.
- Use the listed misconception as evidence about the student's current model.
- Protect the central idea: Subtraction can represent distance, not only taking away.

Answer key and look-fors

1. About $\frac{1}{2}$. 2. 24.

5. $\frac{7}{10} - \frac{1}{4} = \frac{9}{20}$. 6. $\frac{11}{12} - \frac{5}{18} = \frac{23}{36}$.

7. Distance is $\frac{1}{24}$. 8. $\frac{13}{12} = 1 \frac{1}{12}$ m.

Error: $\frac{2}{2} = 1$ exceeds the minuend; correct difference $\frac{11}{24}$.

Exit: $\frac{11}{24}$, between $\frac{1}{3}$ and $\frac{1}{2}$.

MISCONCEPTION TO WATCH

Subtraction can represent distance, not only taking away.

EVIDENCE TO COLLECT

A correct result, a reasonable magnitude check, and an explanation that connects the representation to the symbols.

Instructional focus

Use decomposition, regrouping, or improper fractions strategically.

Primary representation: Number line and equations

Suggested 50-minute arc

5 min	Cumulative retrieval and estimate
8 min	Launch the central comparison or context
12 min	Student modeling and partner reasoning
10 min	Connect representation, language, and notation
10 min	Focused practice with immediate feedback
5 min	Exit evidence: computation plus explanation

Facilitation notes

- Ask students to estimate or predict before applying a procedure.
- Require the referent whole and the fractional unit to remain explicit.
- Use the listed misconception as evidence about the student's current model.
- Protect the central idea: Regrouping one whole adds denominator-sized fractional units.

Answer key and look-fors

1. $6/6$ and $12/12$. 2. About $1 \frac{1}{2}$.
5. $2 \frac{3}{7} + 4 \frac{5}{14} = 6 \frac{11}{14}$. 6. $7 \frac{1}{5} - 3 \frac{3}{4} = 3 \frac{9}{20}$.
7. $1 \frac{5}{6} + 2 \frac{2}{9} = 4 \frac{1}{18}$. 8. $4 \frac{3}{4}$ by direct fractional subtraction.
Error: one whole is $4/4$, so $5 \frac{1}{4} = 4 \frac{5}{4}$.
Exit: $1 \frac{5}{12}$; regroup $4 \frac{1}{6}$ as $3 \frac{7}{6}$.

MISCONCEPTION TO WATCH

Regrouping one whole adds denominator-sized fractional units.

EVIDENCE TO COLLECT

A correct result, a reasonable magnitude check, and an explanation that connects the representation to the symbols.

Instructional focus

Interpret multiplication as repeated groups and as scaling.

Primary representation: Sets, bars, and equations

Suggested 50-minute arc

5 min	Cumulative retrieval and estimate
8 min	Launch the central comparison or context
12 min	Student modeling and partner reasoning
10 min	Connect representation, language, and notation
10 min	Focused practice with immediate feedback
5 min	Exit evidence: computation plus explanation

Facilitation notes

- Ask students to estimate or predict before applying a procedure.
- Require the referent whole and the fractional unit to remain explicit.
- Use the listed misconception as evidence about the student's current model.
- Protect the central idea: The same product can arise from different situations.

Answer key and look-fors

1. $4 \times \frac{3}{4}$. 2. 6.
5. $8 \times \frac{3}{5} = 24/5 = 4 \frac{4}{5}$. 6. $\frac{7}{8}$ of $32 = 28$.
7. $\frac{9}{2} = 4 \frac{1}{2}$ miles. 8. Answers vary with 12 groups of $\frac{5}{6}$.
- Error: "of" signals multiplication; $\frac{3}{4}$ of $20 = 15$.
- Exit: $10/3 = 3 \frac{1}{3}$; ten copies of $\frac{1}{3}$.

MISCONCEPTION TO WATCH

The same product can arise from different situations.

EVIDENCE TO COLLECT

A correct result, a reasonable magnitude check, and an explanation that connects the representation to the symbols.

Instructional focus

Interpret fraction multiplication through nested partitioning.

Primary representation: Area model

Suggested 50-minute arc

5 min	Cumulative retrieval and estimate
8 min	Launch the central comparison or context
12 min	Student modeling and partner reasoning
10 min	Connect representation, language, and notation
10 min	Focused practice with immediate feedback
5 min	Exit evidence: computation plus explanation

Facilitation notes

- Ask students to estimate or predict before applying a procedure.
- Require the referent whole and the fractional unit to remain explicit.
- Use the listed misconception as evidence about the student's current model.
- Protect the central idea: Multiplication does not always make a number larger.

Answer key and look-fors

1. About $\frac{2}{5}$. 2. No.
5. $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$. 6. $\frac{5}{6} \times \frac{3}{10} = \frac{1}{4}$.
7. The product is half of $\frac{7}{8}$, so smaller. 8. $\frac{2}{3}$ of $\frac{3}{2} = 1$.
- Error: multiplying by a proper fraction scales down.
- Exit: $\frac{3}{10}$; both factors are below 1.

MISCONCEPTION TO WATCH

Multiplication does not always make a number larger.

EVIDENCE TO COLLECT

A correct result, a reasonable magnitude check, and an explanation that connects the representation to the symbols.

Lesson 35 | Deriving the Multiplication Procedure

Teacher Guide

Instructional focus

Generalize why numerators and denominators multiply.

Primary representation: Area model and symbolic generalization

Suggested 50-minute arc

5 min	Cumulative retrieval and estimate
8 min	Launch the central comparison or context
12 min	Student modeling and partner reasoning
10 min	Connect representation, language, and notation
10 min	Focused practice with immediate feedback
5 min	Exit evidence: computation plus explanation

Facilitation notes

- Ask students to estimate or predict before applying a procedure.
- Require the referent whole and the fractional unit to remain explicit.
- Use the listed misconception as evidence about the student's current model.
- Protect the central idea: The procedure must preserve the meaning of “of.”

Answer key and look-fors

1. $\frac{2}{5}$. 2. 15 cells.

5. $\frac{15}{56}$. 6. $\frac{2}{15}$.

7. $\frac{4}{5}$. 8. ac/bd.

Error: crossing b rows and d columns creates bd equal cells.

Exit: 15 comes from 3×5 equal cells; numerators count 2×4 overlap cells.

MISCONCEPTION TO WATCH

The procedure must preserve the meaning of “of.”

EVIDENCE TO COLLECT

A correct result, a reasonable magnitude check, and an explanation that connects the representation to the symbols.