

Fraction Fix

Student Workbook | Lessons 31-35

Meaning first. Efficient procedures from understanding. Durable fluency through retrieval.

Lesson progression

31 Subtracting Unlike Fractions

32 Mixed Number Addition and Subtraction

33 Fraction Times a Whole Number

34 A Fraction of a Fraction

35 Deriving the Multiplication Procedure

Working habits

Estimate before computing. Name the unit. Connect a representation to the equation. Check reasonableness.

Name: _____

Lesson 31 | Subtracting Unlike Fractions

1 of 3

Today's Mathematics Interpret subtraction as take-away and distance, then compute with common units.

Essential Question How can one expression describe both removal and distance?

Part I | Get Started

1. Estimate $5/6 - 3/8$.

2. Find the common denominator for sixths and eighths.

Notice and Predict

3. Describe $3/4 - 2/5$ as removal.

4. Describe the same expression as distance between two points.

Sketch, model, or estimate

Lesson 31 | Subtracting Unlike Fractions

2 of 3

Part II | Investigate

Solve paired contexts that use the same expression as removal and comparison.

Representation workspace | Number line and strips



5. Model $\frac{3}{4} - \frac{2}{5}$ on the number line.

6. Compute $\frac{7}{8} - \frac{1}{6}$ and check the sign.

7. Explain how an estimate bounds the exact difference.

Conjecture, connection, or revised model

Part III | Make Sense and Practice

WORKED CONNECTION

Example: $\frac{5}{6} - \frac{3}{8} = \frac{20}{24} - \frac{9}{24} = \frac{11}{24}$, which lies between $\frac{1}{3}$ and $\frac{1}{2}$.

8. Compute $\frac{7}{10} - \frac{1}{4}$.

9. Compute $\frac{11}{12} - \frac{5}{18}$.

10. Find the distance between $\frac{2}{3}$ and $\frac{5}{8}$.

11. A $1\frac{1}{2}$ m ribbon loses $\frac{5}{12}$ m. How much remains?

Error Analysis

12. A student subtracts $\frac{5}{6} - \frac{3}{8}$ by writing $\frac{2}{2}$. Use magnitude to show why this is impossible.

Exit Ticket

- A. Compute $\frac{5}{6} - \frac{3}{8}$.

- B. Give two benchmark bounds for the result.

Lesson 32 | Mixed Number Addition and Subtraction

1 of 3

Today's Mathematics Use decomposition, regrouping, or improper fractions strategically.

Essential Question Which method makes the structure of a mixed-number problem clearest?

Part I | Get Started

1. Rename 1 as sixths and twelfths.

2. Estimate $4\frac{1}{6} - 2\frac{3}{4}$.

Notice and Predict

3. Why does $4\frac{1}{6}$ need regrouping before subtracting $2\frac{3}{4}$?

4. Predict the result's whole-number interval.

Sketch, model, or estimate

Lesson 32 | Mixed Number Addition and Subtraction

2 of 3

Part II | Investigate

Solve examples designed to favor different methods and compare efficiency.

Representation workspace | Number line and equations



5. Solve $3\frac{2}{5} + 1\frac{3}{10}$ by decomposing.

6. Solve $5\frac{1}{4} - 2\frac{5}{6}$ by regrouping.

7. Solve $2\frac{7}{8} + 3\frac{5}{12}$ using improper fractions.

Conjecture, connection, or revised model

Part III | Make Sense and Practice

WORKED CONNECTION

Example: $4 \frac{1}{6} = 3 \frac{7}{6}$; then $3 \frac{7}{6} - 2 \frac{3}{4} = 1 + \frac{14}{12} - \frac{9}{12} = 1 \frac{5}{12}$.

8. Compute $2 \frac{3}{7} + 4 \frac{5}{14}$.

9. Compute $7 \frac{1}{5} - 3 \frac{3}{4}$.

10. Compute $1 \frac{5}{6} + 2 \frac{2}{9}$.

11. Choose the most efficient method for $6 \frac{7}{8} - 2 \frac{1}{8}$.

Error Analysis

12. A student borrows one and changes $5 \frac{1}{4}$ into $4 \frac{2}{4}$. Explain why the new name is not equivalent.

Exit Ticket

- A. Solve $4 \frac{1}{6} - 2 \frac{3}{4}$.

- B. Explain the regrouping using $\frac{6}{6}$.

Today's Mathematics Interpret multiplication as repeated groups and as scaling.

Essential Question How can two contexts share a product but emphasize different meanings?

Part I | Get Started

1. Write $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ as multiplication.

2. Find $\frac{1}{5}$ of 30.

Notice and Predict

3. Predict $\frac{3}{4}$ of 20.

4. How is that context different from 20 groups of $\frac{3}{4}$?

Sketch, model, or estimate

Part II | Investigate

Represent $\frac{3}{4}$ of 20 and 20 groups of $\frac{3}{4}$, then compare meanings and results.

Representation workspace | Sets, bars, and equations

3ths

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4ths

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6ths

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5. Model 5 groups of $\frac{2}{3}$.

6. Model $\frac{2}{3}$ of 15.

7. Connect $n(\frac{a}{b})$, $(\frac{a}{b})n$, and na/b .

Conjecture, connection, or revised model

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Part III | Make Sense and Practice

WORKED CONNECTION

Example: $5 \times \frac{2}{3} = \frac{10}{3} = 3 \frac{1}{3}$. Five groups of two thirds contain ten one-third units.

8. Compute $8 \times \frac{3}{5}$.

9. Find $\frac{7}{8}$ of 32.

10. A runner completes 6 laps of $\frac{3}{4}$ mile. Find the distance.

11. Write a context for $12 \times \frac{5}{6}$.

Error Analysis

12. A student says $\frac{3}{4}$ of 20 equals $\frac{23}{4}$ because “of” means add. Correct the operation and model.

Exit Ticket

- A. Model and compute $5 \times \frac{2}{3}$.

- B. Interpret the numerator and denominator in the product.

Today's Mathematics Interpret fraction multiplication through nested partitioning.

Essential Question How does an area model show a fraction of a fraction?

Part I | Get Started

1. Estimate $\frac{2}{3}$ of $\frac{3}{5}$.

2. Decide whether the product should exceed $\frac{3}{5}$.

Notice and Predict

3. Shade $\frac{3}{5}$ of a rectangle.

4. Now identify $\frac{2}{3}$ of the shaded region.

Sketch, model, or estimate

Part II | Investigate

Find $\frac{2}{3}$ of $\frac{3}{5}$ of a rectangle and track both partitions.

Representation workspace | Area model

5. Partition the grid into fifths in one direction and thirds in the other.

6. Count the overlap and the total equal cells.

7. Explain why the overlap represents the product.

Conjecture, connection, or revised model

Part III | Make Sense and Practice

WORKED CONNECTION

Example: $\frac{2}{3} \times \frac{3}{5}$ gives 6 overlapping cells out of 15, so the product is $\frac{6}{15} = \frac{2}{5}$.

8. Compute $\frac{3}{4} \times \frac{2}{5}$ with a model.

9. Compute $\frac{5}{6} \times \frac{3}{10}$.

10. Compare $\frac{7}{8} \times \frac{1}{2}$ with $\frac{7}{8}$.

11. Find $\frac{2}{3}$ of $1 \frac{1}{2}$.

Error Analysis

12. A student expects $\frac{4}{5} \times \frac{3}{4}$ to be greater than $\frac{4}{5}$ because multiplication makes numbers larger. Diagnose the whole-number assumption.

Exit Ticket

- A. Draw and compute $\frac{3}{4} \times \frac{2}{5}$.

- B. Explain why the product is smaller than both factors.

Today's Mathematics Generalize why numerators and denominators multiply.

Essential Question Why does a fraction-by-fraction area model create ac parts out of bd?

Part I | Get Started

1. Compute $\frac{2}{3} \times \frac{3}{5}$ from the prior model.

2. How many equal cells appear when thirds cross fifths?

Notice and Predict

3. Predict the number of overlap cells for $\frac{2}{3} \times \frac{4}{5}$.

4. Predict the total number of equal cells.

Sketch, model, or estimate

Part II | Investigate

Label a generic partitioned rectangle and connect subareas to ac/bd .

Representation workspace | Area model and symbolic generalization

5. Represent $a/b \times c/d$ on a rectangle.

6. Explain why the overlap count is ac .

7. Explain why the whole is partitioned into bd cells.

Conjecture, connection, or revised model

Part III | Make Sense and Practice

WORKED CONNECTION

Example: $\frac{2}{3} \times \frac{4}{5} = \frac{(2 \times 4)}{(3 \times 5)} = \frac{8}{15}$. The denominator 15 counts the 3-by-5 grid cells.

8. Compute $\frac{5}{7} \times \frac{3}{8}$.

9. Compute $\frac{4}{9} \times \frac{3}{10}$ and simplify.

10. Use factors to simplify $\frac{6}{11} \times \frac{22}{15}$.

11. Write a general equation for $\frac{a}{b} \times \frac{c}{d}$.

Error Analysis

12. A student multiplies numerators but adds denominators. Use the area grid to show why the total cell count is a product.

Exit Ticket

- A. Explain the denominator in $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

- B. State the general multiplication rule with meaning.
