

Fraction Fix

Student Workbook | Lessons 36-40

Meaning first. Efficient procedures from understanding. Durable fluency through retrieval.

Lesson progression

36 Scaling and Reasonableness

37 Mixed Numbers and Cross-Cancellation

38 Two Meanings of Division

39 Whole Number Divided by a Unit Fraction

40 Unit Fraction Divided by a Whole Number

Working habits

Estimate before computing. Name the unit. Connect a representation to the equation. Check reasonableness.

Name: _____

Today's Mathematics Predict the effect of multiplying by values below, equal to, or above one.

Essential Question How does the size of a scale factor predict the product?

Part I | Get Started

1. Classify $\frac{5}{6}$ as below, equal to, or above 1.

2. Predict whether $8 \times \frac{5}{6}$ is below or above 8.

Notice and Predict

3. Compare $\frac{7}{8} \times \frac{5}{6}$ with $\frac{7}{8}$ without calculating.

4. What would multiplying by 1 do?

Sketch, model, or estimate

Lesson 36 | Scaling and Reasonableness

2 of 3

Part II | Investigate

Classify products as greater than, equal to, or less than the starting quantity before calculating.

Representation workspace | Number line and scale factors



5. Place factors $1/2$, 1, and $3/2$ on a scale-factor line.

6. Predict the effect of each on $4/5$.

7. Check one prediction by computing.

Conjecture, connection, or revised model

Part III | Make Sense and Practice

WORKED CONNECTION

Example: because $\frac{5}{6} < 1$, multiplying $\frac{7}{8}$ by $\frac{5}{6}$ produces a positive result less than $\frac{7}{8}$.

8. Compare $\frac{9}{10} \times \frac{7}{4}$ with $\frac{9}{10}$.

9. Compare $\frac{11}{6} \times 1$ with $\frac{11}{6}$.

10. Order $\frac{3}{4} \times \frac{1}{2}$, $\frac{3}{4} \times 1$, and $\frac{3}{4} \times 2$.

11. Choose a factor that doubles $\frac{5}{8}$.

Error Analysis

12. A student says $\frac{2}{3} \times \frac{3}{4}$ must exceed both factors because 2×3 and 3×4 are larger. Explain the flawed comparison.

Exit Ticket

- A. Without calculating, compare $\frac{7}{8} \times \frac{5}{6}$ with $\frac{7}{8}$.

- B. State the scale-factor principle used.

Today's Mathematics Multiply efficiently while preserving factor meaning.

Essential Question How can factor structure simplify multiplication before numbers grow?

Part I | Get Started

1. Rename $2\frac{1}{4}$ and $1\frac{2}{3}$ as improper fractions.

2. Estimate their product.

Notice and Predict

3. Identify common factors across $\frac{9}{4} \times \frac{5}{3}$.

4. Explain what cancellation does to the product value.

Sketch, model, or estimate

Part II | Investigate

Compare conversion-first, distributive, and factor-cancellation approaches.

Representation workspace | Equations and area decomposition

5. Compute $2\frac{1}{4} \times 1\frac{2}{3}$ by cancellation.
6. Compute the same product using distribution.
7. Compare the efficiency and visibility of the methods.

Conjecture, connection, or revised model

Part III | Make Sense and Practice

WORKED CONNECTION

Example: $9/4 \times 5/3 \rightarrow 3/4 \times 5/1 = 15/4 = 3 \frac{3}{4}$ after dividing 9 and 3 by 3.

8. Compute $3 \frac{1}{2} \times 2 \frac{2}{7}$.

9. Compute $4/15 \times 25/8$ efficiently.

10. Compute $1 \frac{3}{5} \times 2 \frac{1}{4}$.

11. Explain why cancellation cannot occur across addition.

Error Analysis

12. A student cancels the 3s in $(3+5)/3$. Explain why the numerator is a sum, not a factor.

Exit Ticket

- A. Compute $2 \frac{1}{4} \times 1 \frac{2}{3}$ and estimate first.

- B. Name the factors canceled and the equivalent product.

Today's Mathematics Distinguish sharing division from measurement division.

Essential Question Does division ask the size of each share or the number of groups?

Part I | Get Started

1. Write a sharing story for 12 divided by 3.

2. Write a measurement story for the same expression.

Notice and Predict

3. What question does 3 divided by $\frac{1}{2}$ ask?

4. Why is “share among one-half people” not a useful context?

Sketch, model, or estimate

Part II | Investigate

Write two stories for 3 divided by $\frac{1}{2}$ and identify the measurement interpretation.

Representation workspace | Counters, strips, and equations

3ths

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4ths

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6ths

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5. Represent 3 divided by $\frac{1}{2}$ as equal jumps.

6. Represent $\frac{3}{4}$ divided by 3 as sharing.

7. Label the units of each quotient.

Conjecture, connection, or revised model

Part III | Make Sense and Practice

WORKED CONNECTION

Example: 3 divided by $\frac{1}{2}$ asks how many half-unit groups fit in 3; six groups fit.

8. Write and solve a measurement story for 4 divided by $\frac{2}{3}$.

9. Write and solve a sharing story for $\frac{2}{3}$ divided by 4.

10. Classify 5 divided by $\frac{1}{4}$.

11. Classify $\frac{3}{5}$ divided by 3.

Error Analysis

12. A student interprets every division problem as fair sharing. Use 4 divided by $\frac{2}{3}$ to show the limitation.

Exit Ticket

- A. Write and solve a measurement story for 4 divided by $\frac{2}{3}$.

- B. Identify dividend, divisor, quotient, and units.

Lesson 39 | Whole Number Divided by a Unit Fraction

1 of 3

Today's Mathematics Interpret division as counting how many fractional units fit.

Essential Question Why can division by a number below one produce a larger quotient?

Part I | Get Started

1. How many fourths make 1?

2. How many fourths make 3?

Notice and Predict

3. Predict 5 divided by $\frac{1}{3}$.

4. Describe what the quotient counts.

Sketch, model, or estimate

Lesson 39 | Whole Number Divided by a Unit Fraction

2 of 3

Part II | Investigate

Determine how many $\frac{1}{4}$ -cup servings fit in several whole-cup amounts.

Representation workspace | Measurement strips and number line



5. Show 3 divided by $\frac{1}{4}$ as jumps.

6. Connect 3 divided by $\frac{1}{4}$ to 3 times 4.

7. Generalize n divided by $\frac{1}{b}$.

Conjecture, connection, or revised model

Part III | Make Sense and Practice

WORKED CONNECTION

Example: 3 divided by $\frac{1}{4} = 12$ because each whole contains four fourths and three wholes contain twelve.

8. Compute 7 divided by $\frac{1}{2}$.

9. Compute 4 divided by $\frac{1}{5}$.

10. A baker has 6 cups and uses $\frac{1}{3}$ cup per batch. Find the batches.

11. Write a context for 2 divided by $\frac{1}{8}$.

Error Analysis

12. A student says 5 divided by $\frac{1}{3}$ should be smaller than 5. Use groups to correct the claim.

Exit Ticket

- A. Model 5 divided by $\frac{1}{3}$.

- B. Explain why the quotient is 15.

Today's Mathematics Interpret equal sharing of a fractional amount.

Essential Question How does sharing one fractional unit create a smaller unit?

Part I | Get Started

1. Split one whole into sixths.

2. How many sixths equal $\frac{1}{2}$?

Notice and Predict

3. Predict $\frac{1}{2}$ divided by 3.

4. What new unit is created by the sharing?

Sketch, model, or estimate

Part II | Investigate

Share $\frac{1}{2}$ meter equally among 3 people and identify each share.

Representation workspace | Strips and sharing diagrams

3ths

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4ths

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6ths

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5. Partition a half into 3 equal shares.

6. Connect the share to $\frac{1}{6}$ of the original whole.

7. Generalize $\frac{1}{b}$ divided by n .

Conjecture, connection, or revised model

Part III | Make Sense and Practice

WORKED CONNECTION

Example: $\frac{1}{2}$ divided by 3 = $\frac{1}{6}$ because each of three people receives one of six equal parts of the whole.

8. Compute $\frac{1}{4}$ divided by 3.

9. Compute $\frac{1}{5}$ divided by 4.

10. Share $\frac{1}{3}$ liter among 5 people.

11. Compare $\frac{1}{6}$ divided by 2 with $\frac{1}{12}$.

Error Analysis

12. A student says $\frac{1}{4}$ divided by 3 equals $\frac{3}{4}$ because division makes the numerator larger. Use a strip to correct the model.

Exit Ticket

- A. Model and compute $\frac{1}{4}$ divided by 3.

- B. Explain how the denominator changes.
